David Corfield, *Towards a Philosophy of Real Mathematics*. Cambridge: Cambridge University Press (2003), x+288 pp., $70.00 (cloth).

The primary goal of the book under review is to reorient the philosophy of mathematics away from what is often called the foundations of mathematics and towards the "real mathematics" of the working mathematician. Along with this shift in what parts of mathematics would occupy the philosopher's attention, Corfield argues for introducing a new collection of questions into the philosophy of mathematics that are supposed to arise more naturally in real mathematics. Such a revolutionary program will no doubt create some uneasiness in those schooled in the intricacies of contemporary philosophy of mathematics, and their concerns will be heightened by Corfield's frequent invocation of the philosophy of the special sciences, like physics. For, like current philosophy of physics where "If you want to be a philosopher of physics, you simply must be conversant with" theories like quantum field theory (235), Corfield envisions a philosophy of mathematics where knowledge of algebraic topology and even more recent innovations like higher-dimensional category theory is presupposed. While Corfield does not succeed in arguing that such investigations should replace philosophical investigations into the foundations of mathematics, he impressively demonstrates the rich new philosophical work that should be done "informed by the concerns of mathematicians past and present" (3).

After a long introduction summarizing his new approach to the philosophy of mathematics, Corfield divides the book into four different parts outlining four new or often neglected topics for the philosopher to ask about non-foundational mathematics. In "Part I: Human and Artificial Mathematics" the focus is on using computers to discover proofs of open questions, to improve on proofs of known theorems and even to propose new conjectures for mathematical investigation. While the results achieved by computer scientists to date are not too earth shattering, Corfield teases out a number of philosophical issues from this contemporary research. Interestingly, he uses this work in computer science to motivate a discussion of the role of analogy in the development of new mathematics, taking as his "watershed" moment Dedekind and Weber's joint paper of 1882 which "laid the foundations for an extraordinarily rich transfer of concepts between the fields of algebraic number theory and algebraic function theory" (96). "Part II: Plausibility, Uncertainty and Probability" and "Part III: The Growth of Mathematics" are extended reflections on Pólya and Lakatos, respectively. Pólya's discussions of mathematical reasoning are updated using Bayesian machinery, although Corfield argues that Bayesian approaches to science must also be adjusted if they are to handle mathematics and the use of mathematics in science. Though critical of many of the details of Lakatos' own work on the development of mathematics, Corfield uses it as a basis to outline his own model of mathematical research. This sets up one of the most interesting chapters in the book on the relative merits of groups versus groupoids, a more general sort of mathematical construction which includes groups as a special case.[1] Corfield investigates the resistance that some feel to working with groupoids and the responses offered by groupoid advocates. Among the topics raised are the naturalness of the connections that groupoids forge with other areas of mathematics and whether or not the groupoid definition was an inevitable mathematical discovery.

[1] The most compact definition of a groupoid is in category theory: "a groupoid is just a small category in which every arrow is invertible … From this perspective, groups … are groupoids with only one object" (211).

This sort of careful presentation of mathematical research and debate is repeated in "Part IV: The Interpretation of Mathematics", an extended discussion of higher-dimensional algebra, also known as higher-dimensional category theory. Corfield conjectures that this fairly recent mathematical theory will occupy a central place in the mathematics of this century and at one point, extending a suggestion by Atiyah, offers the following scheme: "Nineteenth century: The study of functions of one variable. Twentieth century: The study of functions of many variables. The search for structural features captured by 1-category theory. Twenty-first century: The search for structural features captured by *n*-category theory" (249). The mathematical argument that higher-dimensional category theory will be so important is difficult to summarize, but rests largely on the idea that this theory provides an insightful interpretation of large parts of contemporary mathematics. "Interpretation" is meant here not in the sense in which one might give a platonistic or nominalistic interpretation of mathematics, but rather as a recasting of familiar mathematical objects in new terms. An influential interpretation is a set-theoretic interpretation of mathematics, which identifies standard mathematical objects like the real numbers with particular sets. However, Corfield rejects such an interpretation as mathematically unilluminating, and opposes the "decategorification" needed for a set-theoretic interpretation with the "categorification" whose generalization leads to higher-dimensional category theory (241).

Corfield's focus on difficult contemporary mathematics and the novel philosophical questions that it raises is certainly to be praised, but it seems that in his eagerness to introduce philosophers to real mathematics he has failed to engage with the motivations that many have for working on more traditional topics in the philosophy of mathematics. Building on his appeal to the philosophy of the special sciences, we can note that despite the increased focus on the philosophy of physics, biology, chemistry, etc. hardly anybody is advocating that we stop working on the general philosophical issues raised by science like realism or reduction. Similarly, even if new philosophical concerns arise as a result of the study of algebraic topology, this in itself is no reason to set aside old philosophical debates like platonism versus nominalism and the connection between mathematics and logic, or to abandon philosophical investigations into set theory. Perhaps such work is less engaged with the priorities of the working mathematician, but it is of great importance for another community that should occupy at least some of the philosopher of mathematics' attention: the working metaphysician and epistemologist. Metaphysics and epistemology depend on the understanding of mathematics that foundational work can provide, just as they rely on general philosophy of science. So, while I am sympathetic to Corfield's argument that a wider variety of mathematics and philosophical questions should be studied in the philosophy of mathematics, I must reject the suggestion that foundational mathematics is not real mathematics worthy of continued philosophical investigation.

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