On Batterman's "On the Explanatory Role of Mathematics in Empirical Science"

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Abstract

This discussion note of (Batterman [2010]) clarifies the modest aims of my 'mapping account' of applications of mathematics in science. Once these aims are clarified it becomes clear that Batterman's 'completely new approach' (Batterman [2010], p. 24) is not needed to make sense of his cases of idealized mathematical explanations. Instead, a positive proposal for the explanatory power of such cases can be reconciled with the mapping account.

Batterman's work on the way in which asymptotic techniques are deployed in science has been a significant influence on my own thinking about the role of mathematics in science, so it is a pleasure to see him turning his focus squarely on the question of how mathematics helps in explanation. At the same time, I want to clarify the aims of my original mapping approach to the content of mathematical claims and indicate the modest aims of my discussion of mathematical idealization. This will clear the way for a discussion of how a mapping account can be combined with a positive proposal for the explanatory power of asymptotic techniques. So, far from it being the case that 'a completely new approach is needed' (Batterman [2010], p. 24), it will turn out that the discussion of explanatory power can build on my earlier discussions of mappings and idealization. I do not want to suggest that this approach to explanation was already implicit in the mapping account, but only that the two views are compatible.

A further qualification is also necessary. In his paper Batterman engages with other views on mathematics and explanation, especially Bueno and Colyvan's inferential conception (Bueno and Colyvan [forthcoming]). It would be a delicate matter to determine how my mapping account relates to the inferential conception and how the inferential conception proposes to make sense of mathematical explanations in science. I will set those issues aside here, although I hope that the distinction I emphasize between description and explanation is of interest to all participants in this debate.

There is a longstanding distinction in the history and philosophy of science between description and explanation. As I make clear in several places, my primary focus was on assigning descriptive contents to mixed mathematical claims as well as scientific models whose specification requires mathematics. For example, in my first article on the issue I say 'we can sharpen the issue by focusing on the content or meaning of the statements that reflect our reasoning in these situations' (Pincock [2004], p. 137). In a later discussion I emphasize that 'it is in terms of mappings that the statements of applied mathematics are assigned appropriate truth-conditions' (Pincock [2007b], p. 262). And in the third paper which Batterman cites I discuss how to 'fix the content of a scientific representation using both the features of the mathematical models and the contents of the beliefs and intentions of the agents doing the representing' (Pincock [2007a], p. 959). These contents should clarify under what circumstances the claim or model is true of a given target system. In more complicated situations, the content should allow that the model is true of one aspect of a target system even when it is false of another aspect of that system.[1] This is what I intend a mapping account to accomplish. Very briefly, for any mixed mathematical claim or scientific model, one must indicate (i) what purely mathematical entity or structure is in question, (ii) how the parts of the mathematics are to be physically interpreted and (iii) what sort of structural relation must obtain between the interpreted mathematical structure and the target system for the claim or model to be accurate. Let us call this task the assignment of a representational content to a claim or model. Once this representational content is specified, it becomes possible to make sense of our reasoning with that claim or model, including activities of prediction, confirmation and acceptance.

Batterman argues that this mapping approach to content is not sufficient to account for the explanatory power of some idealized representations. I agree. This is why I said 'even if we have a satisfactory account of the meaning of mixed statements there might still be a further problem of explaining the success of applications of mathematics' (Pincock [2004], p. 137, fn. 4). Part of explaining this success is showing how mathematics contributes explanatory power. But I continue to believe that 'clarifying the meaning of mixed statements is a necessary first step' in addressing these other problems (Pincock [2004], p. 137, fn. 4). More generally, we should not expect an account of how mathematics describes a target system to be able to provide a complete account of how mathematics can be used to explain features of the target system. This is because explanation usually requires more than merely accurate description. This point is consistent with the use of the

[1] See Pincock [forthcoming].

mapping account to ground some ways in which mathematics can help in explanation. In the Königsberg bridges case, the explanatory power is tied to the simple way in which the model abstracts from the irrelevant details of the target system. It throws out what is irrelevant and highlights what is relevant. Crucially, what is relevant is the mathematical structure found in the target system itself. So, here at least is one case where an account of content lends itself naturally to grounding explanatory power. But I explicitly noted in my discussion of the bridges case that this sort of 'abstract explanation' is different from Batterman's cases: 'some abstract explanations are not asymptotic explanations ... [and] abstract explanations generally require philosophical examination' (Pincock [2007b], p. 260). There is no suggestion that I took the mapping account of content by itself to be sufficient to account for Batterman's cases of explanation.

In my paper on idealization I distinguish between good and bad idealizations. This distinction draws on the mapping account of content, but also appeals to other factors like the beliefs and goals of the scientists which deploy the model. Batterman correctly takes this attempt to distinguish good and bad idealizations to involve 'an account of how one can in fact *rank* idealizations (with their false assumptions) as better or worse for representing the world' (Batterman [2010], p. 11, my emphasis). For example, I try to 'provide the ingredients sufficient to rank idealized representations in various respects' (Pincock [2007a], p. 961). But it is important to be clear that I did not intend to offer such a ranking of idealizations only in terms of their goodness. There is a second respect in which idealizations are ranked. This is in terms of our knowledge of the goodness of these idealizations (Pincock [2007a], §4). Here I consider three sorts of cases in terms of 'epistemic security' (Pincock [2007a], p. 963). This is valuable because an idealization may be good and we may fail to know this. But, contrary to what Batterman suggests, I do not claim that my account of idealization offers enough to make sense of the explanatory power of scientific models, including those pertaining to phase transitions and supernumerary bows. An idealization's goodness, either in the sense that it accomplishes the purposes of scientists or in the sense that we know that it does so, may have little to do with its explanatory power.

I conclude that, at best, what Batterman has shown is that a natural way of extending the mapping account to accomplish the further task of providing an account of explanation runs into problems. I agree, but want to briefly sketch another way of adding to the mapping account of content which avoids the problems that Batterman raises. The basic idea is that an explanation can depend upon the mathematical relations between two scientific models. For Batterman's cases, the mathematical relation is that model B results from model A by asymptotic reasoning. But this is consistent with the content of model A and the content of model B being analyzed in terms of the three components of the mapping account to content outlined above. This allows the contents delivered by the mapping account to contribute to our reconstruction of the source of the explanatory power of the appeal to model B. I will outline how this works with two cases corresponding to Batterman's emphasis on the distinction between regular and singular limits. A regular limit is involved in an explanation of wave dispersion, while a singular limit appears in the explanation of the pattern of spacing of the supernumerary bows. Given the nature of a discussion note, it is not feasible to present these cases in the level of detail necessary to show that the account of explanation I suggest is the best one available. However, I believe that my summaries are sufficient to indicate a strategy for reconciling my account of content with the account of explanation suggested by Batterman. As Batterman has argued that these accounts are incompatible, even these outlines should prove valuable in moving the debate forward.

When a rock is dropped in an otherwise calm ocean, it produces an irregular disturbance in the surface. In some cases, as the disturbance propagates outwards, it becomes more regular as the waves in the original superposition with a longer wavelength move more quickly than the waves with a shorter wavelength. This is an instance of wave dispersion. To explain it, we start with a suitable version of the Navier-Stokes equations from fluid mechanics. This is our first model A and its content can be assigned using the mapping account. Then we consider the limit where the ratio of the depth of the ocean to the wavelength goes to infinity. This leads to the second model B. The second model is crucial to our explanation because it allows one to derive the following equation for the velocity c of a wave with wavelength λ:

$$c = \sqrt{\frac{g\lambda}{2\pi}} \tag{1}$$

g is the acceleration due to gravity. So, (1) claims that the speed of a wave will increase as its wavelength increases. The pair of models A and B along with the mathematical link in terms of the limit is needed for this explanation to go through.[2]

Even with this simple case we can appreciate Batterman's search for an answer to the question 'How does having a representation or a partial representation of a physical situation in mathematical terms provide an *explanation* of that physical situation?' (Batterman [2010], p. 16). Considering B and its content is not sufficient to recover the explanation. What we need

[2] See Kundu and Cohen [2008], §7.6; Pincock [unpublished (a)], ch. 5, [unpublished (b)] for further details and discussion.

is the mathematical link between A and B because only this allows us to appreciate the relevant features of the target system. The way in which the mathematical link is given is central to the explanation. For what it shows is that the specific depth of the ocean is irrelevant as long as it exceeds a certain threshold. In fact, we can classify waves as deep-water waves when the depth is greater than .28 times the wavelength λ and argue that (1) can explain wave dispersion for all such waves.

Does this case support Batterman's claim that 'limiting operations ... are simply not the sorts of gizmos which figure in a (partial) representation' (Batterman [2010], p. 19) and does this claim undermine the contribution that the mapping account makes to an account of this explanation? Clearly, with respect to the depth of the ocean and model B, that part of the mathematical structure is not representing any feature of the ocean. So, I agree with Batterman's point that limits can 'yield various types of divergences and singularities for which there are *no* physical analogs' (Batterman [2010], p. 19). But we can make sense of this situation by appeal to how the content of model A differs from the content of model B. Model A involves the physical interpretation of a part of the mathematical structure as representing the depth of the ocean. The limit which produces model B from model A removes this physical interpretation from the corresponding part of the mathematical structure of the model B. So, B literally has nothing to say about the depth of the ocean. How does this contribute to the explanation? Taking this limit makes explicit that the depth of the ocean is irrelevant if it surpasses a certain threshold and highlights the other parameters which are relevant to the speed of the wave, namely the wavelength λ and g.

This case involves a regular limit, so nearly all of the physical interpretation of A can be transferred over to the physical interpretation of B, except of course the depth. But Batterman emphasizes the special explanatory powers associated with limits which are singular: 'singularities make it impossible to tell any kind of de-idealizing story' (Batterman [2010], p. 18) of the sort supposedly required by my approach. So, it is worth briefly summarizing how the mapping account of content can contribute to an account of the explanatory power of a case involving a singular limit. A case which Batterman mentions and which I have tried to reconstruct in some detail is the explanation of the existence and spacing of the supernumerary bows. Here there are three models.[3] First, a light wave model A is taken to a light ray model B by considering the limit of the ratio of the wavelength to the drop radius going to 0.[4] Then an intermediate model C is specified in terms of the features of both A and B. Despite these complexities and the singular nature of the limits involved, there is no barrier to reconstructing the contents of models A, B and C using the mapping account. As with the wave dispersion case, the limit which links A to B requires that certain features of light waves are irrelevant to the phenomenon being explained once the parameter in question falls below a certain threshold. The links from A to C and B to C are more subtle as they turn on forms of mathematical reasoning beyond simply taking a limit. The singular limit and these other forms of reasoning do have more dramatic effects on how parts of the mathematics will be reinterpreted in the course of the explanation. But I believe it is possible to reconstruct these sorts of explanations with the aid of the mapping account of content.

I conclude that a focus on asymptotic reasoning provides new opportunities to combine the mapping account of content with a positive account of explanatory power. What we need to appreciate the significance of asymp-

[3]See Nussenzveig [1977], [1992]; Pincock [unpublished (a)], ch. 11, [unpublished (c)].

[4]See Batterman [2010], p. 21, fn. 23 on the need to consider this ratio.

totic reasoning is *both* an account of representational content and a clear proposal for what distinguishes explanations from mere descriptions. The proposal for explanation suggested by Batterman turns on removing irrelevant details and highlighting relevant factors for the phenomenon which is being explained. I have tried to indicate how this account of explanation can benefit from the mapping account of content. It may be that Batterman prefers to develop an alternative account of content. If so, I will look forward to seeing what he comes up with. Without some account of the content of his models, Batterman's proposal risks turning his explanations into purely mathematical derivations. This is obviously not his intention as he has repeatedly emphasized how these explanations help us to learn more about the physical world than would be possible without asymptotic techniques.

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